Finite Element for Fluids - HM5

Arthur Lustman

April 2, 2019

1 Stokes problem

The code describe a Stokes problem in a squared domain of surface 1 confined with the top boundary moving. Which mean that the Dirichlet boundary condition of the velocity is set to be zero on the sides except on top where it is equal to 1. This can be seen in figure 5, representation of the flow computed.

Since the pressure needs at least 1 Dirichlet boundary condition defined so that the problem is well defined: the left corner Dirichlet boundary condition for the pressure is set to 0.

1.1 Code modification

The code works for both quadratic elements Q2Q1 and P2P1 but is not stable for linear elements due to the LBB condition. Stabilization terms needs to be added for them in order for the solution to reach a stable state for those type of elements. The GLS stabilization method uses a $\overline{\tau}$ parameter which is equal to

$$\tau_1 = \alpha_0 \frac{h^2}{4\nu} \qquad \tau_2 = 0 \tag{1}$$

Where $\alpha_0 = 1/3$ is given as optimal for linear elements.

The formulae of the matrices to be added to the general system of equation is described on the slide 34 of the Incompressible flow chapter of the course. But since we are using only linear terms for the stabilization part, and only τ_1 is non zeros, there is only $\boldsymbol{L}$ and $\overline{f_q}$ which are non zeros. For which their formula is given as

$$\boldsymbol{L} \longrightarrow \sum_e \int_{\Omega_e} \tau_1 (\nabla q) \cdot (\nabla p) d\Omega \tag{2}$$

$$\overline{f}_q \longrightarrow \sum_e \int_{\Omega_e} \tau_1 (\nabla q) \cdot (-\overline{f}) d\Omega \tag{3}$$

From there, the expression of these integrals become

$$\boldsymbol{L}_e \longrightarrow \sum_{\text{ngaus}} \tau_1 \begin{bmatrix} N'_x \, N'_y \end{bmatrix} \begin{bmatrix} N'_x \\ N'_y \end{bmatrix} |J| \, w \tag{4}$$

$$\overline{f}_{q_e} \longrightarrow -\sum_{\text{ngaus}} \tau_1 \begin{bmatrix} N'_x N'_y \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix} |J| \, w \tag{5}$$

$$\tag{6}$$

Where N'_x is the x derivative of the shape function, $|J|$ is the determinant of the Jacobian, w is the weight of the gauss point, f_x is the value of the vector $\overline{f}$ in the x component. All those variable are evaluated at the integration point.

1.2 Code results

The results are coherent. The solutions all look similar and look more precise as the mesh is refined, displayed in figure 1 to 4. Those figures represent the value of the pressure at the nodes for P1P1 elements with a different number of nodes in each direction.

The flow can be seen on figure 5, set for 10 nodes in each direction for Q1Q1 elements.

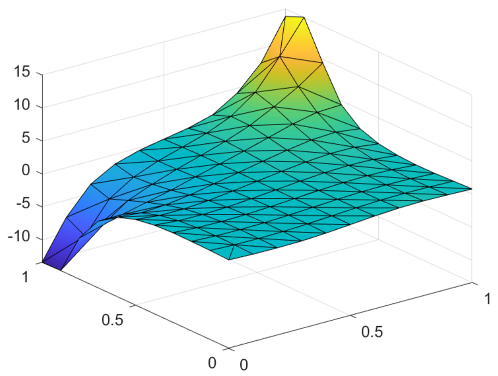

Figure 1: P1P1 with 10 nodes

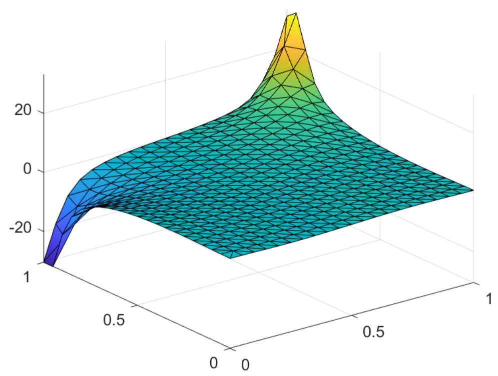

Figure 2: P1P1 with 20 nodes

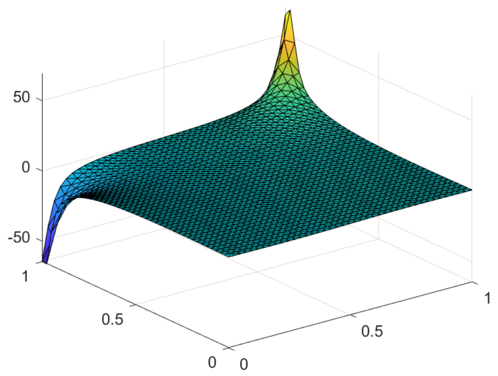

Figure 3: P1P1 with 40 nodes

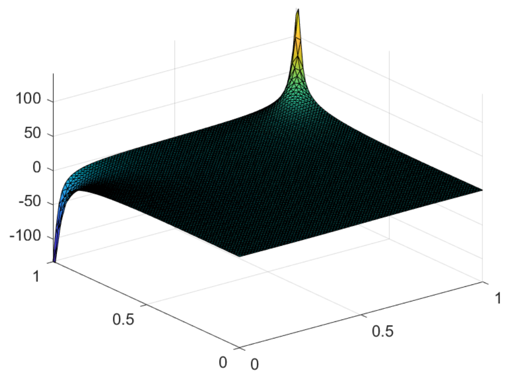

Figure 4: P1P1 with 80 nodes

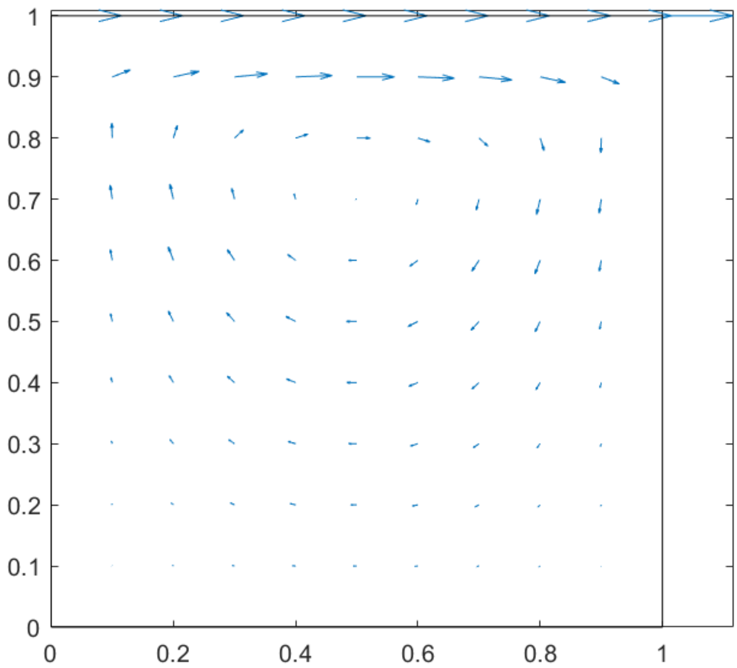

Figure 5: Flow in the domain