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Incompressible Navier-Stokes equations

1. Compute the velocity and pressure errors:

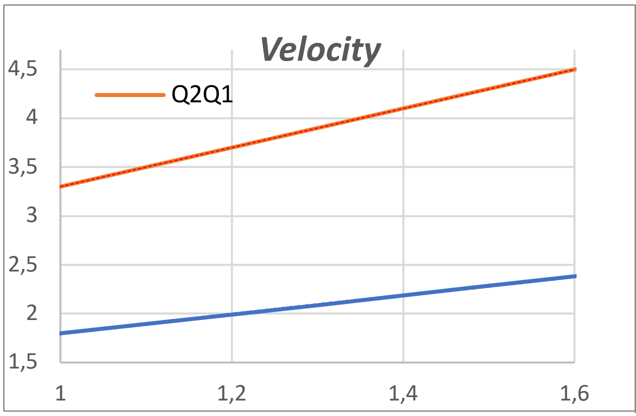
$$e_{\boldsymbol{v}} = \sqrt{\int_{\Omega} (\nabla \boldsymbol{v} - \nabla \boldsymbol{v}^h)^2 d\Omega} \quad \text{and} \quad e_p = \sqrt{\int_{\Omega} (p - p^h)^2 d\Omega}$$

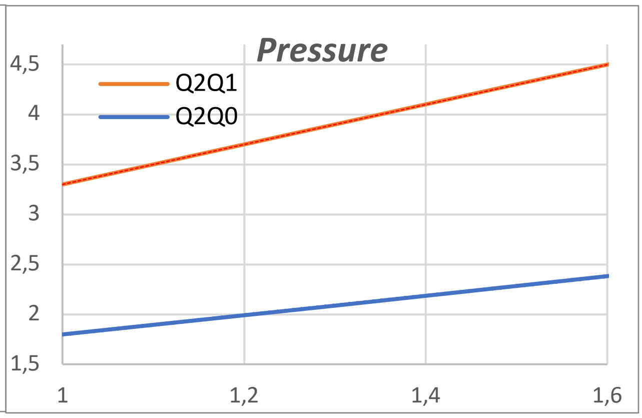
Taking the provided code from the Lab class, where it is implemented already the Stokes problem, we are going to analyze the convergence based on the elements that fulfill the LBB condition for the pressure and velocity space (avoiding instabilities in the pressure field).

a) Check the convergence for Q2Q1 and Q2Q0 elements.
In order to be able to compute the integral over the domain, it is important to perform this computation on the gauss points.
It is necessary provide different mesh sizes in order to track the convergence and the error of the solution.

Number of nodes	10	20	30	40
Mesh size	1/10	1/20	1/30	1/40

Once the necessary datum is provided, we are going to present the solution for the convergence in the following figures:





As it can be seen, the rate of convergence for the elements Q2Q1 is 2, due to the degrees of interpolation of second order for velocity and linear for pressure. In the other hand, for the elements Q2Q0, represent a second order of interpolation for the velocity field and constant for pressure, thus providing a rate of convergence of order 1. Both cases fulfill the LBB condition, providing the positiveness of the matrix (G' K^{-1} G).

b) Solve the problem using P1P1 elements:
The provided P1P1 elements doesn't fulfill the condition LBB, arising oscillations for the pressure field.

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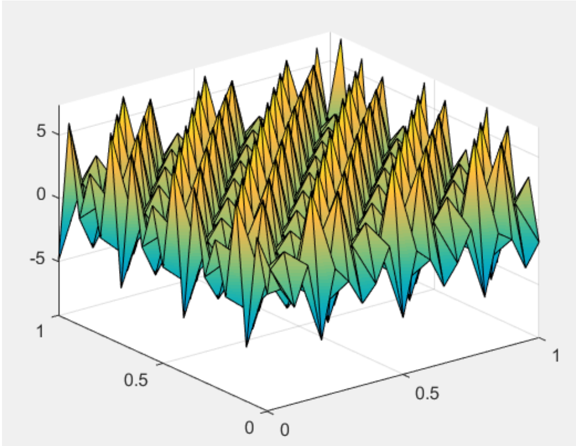

Figure 1 Pressure field

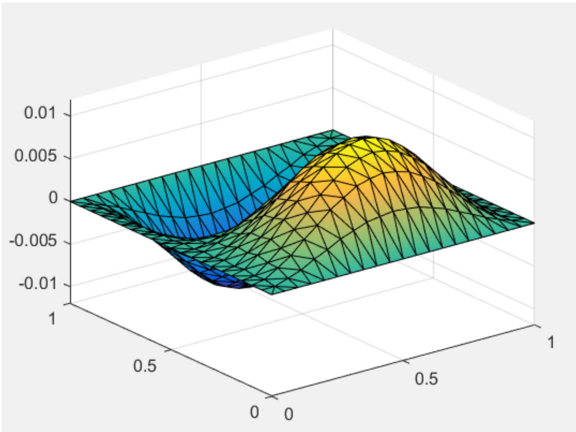

Figure 3 Velocity filed in x direction

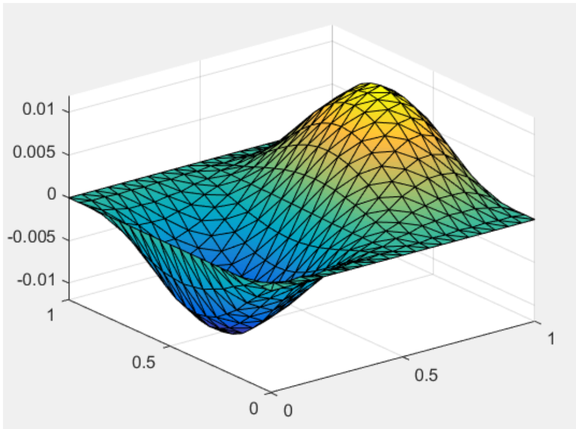

Figure 2 Velocity field in y direction

As it was predicted, the pressure field present oscillation arising from the lack of positiveness of matrix (G’ K^{-1} G).
In order to force the kernel = 0 from the mentioned matrix, it is necessary to introduce the stabilization term as SUPG or GLS.
The stabilized problem is shown, computing a smooth solution for pressure and keeping the same result as before for the velocity field.

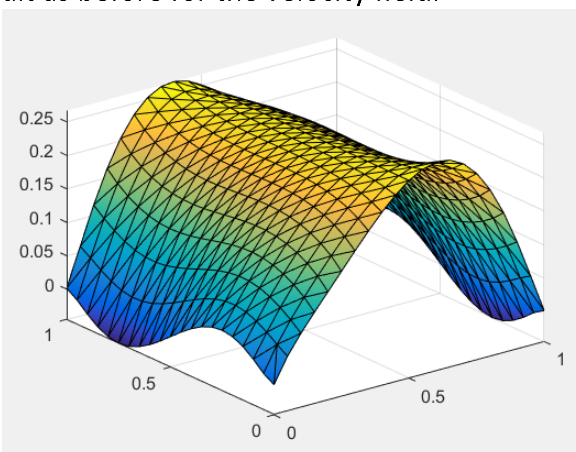

Figure 4 Pressure field after stabilize

2. Provided a structures mesh of 20x20 elements and a mesh of the same singularities refined on the edges, provide a code computing the cavity flow problem discuss the results:

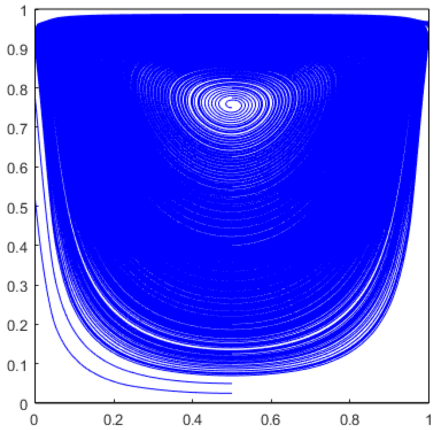

Figure 5 Streamlines for regular mesh

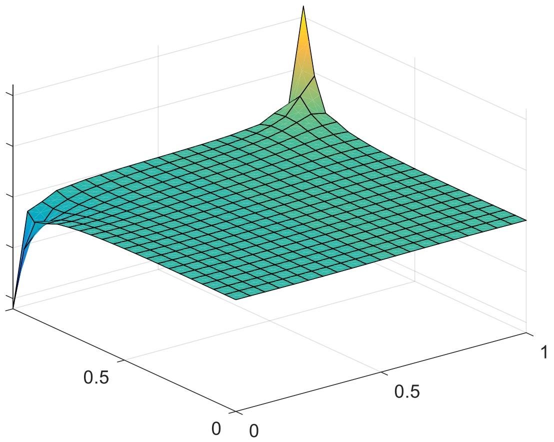

Figure 6 Pressure field for regular mesh

As it can be seen, the plots for the regular mesh it presents not accurate results on the corners, where the gradient is high, presenting no reliable approximation of the problem. From that drawbacks, the need to refine the mesh on the edges arises, solved with the following mesh:

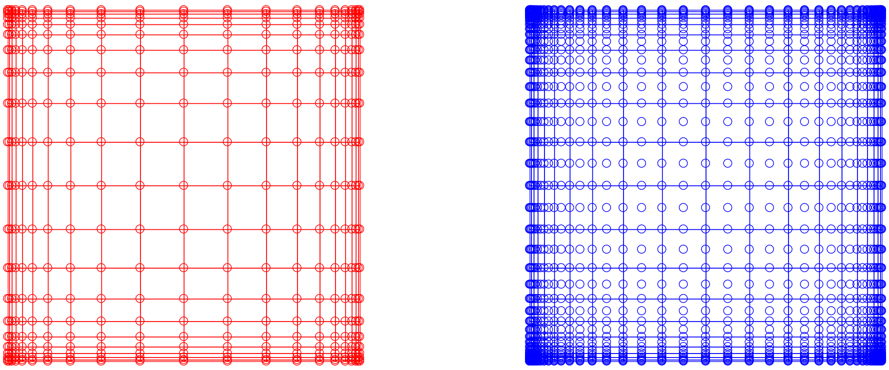

Figure 7 Refined pressure and velocity mesh

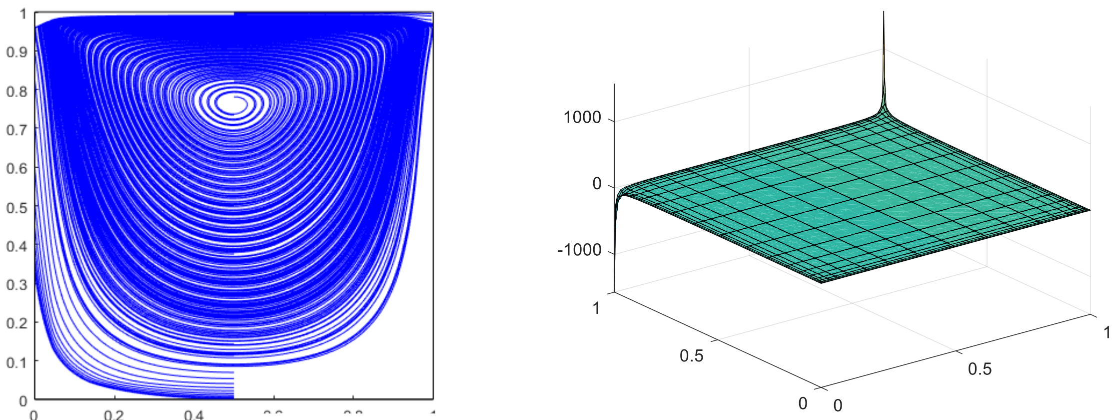

Figure 8 Streamlines for refined mesh

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The results computed with refined mesh it is able to capture the pressure gradient in the upper corners for the pressure field, thus giving a reliable solution.

The same problem is computed for different values of Reynolds number:

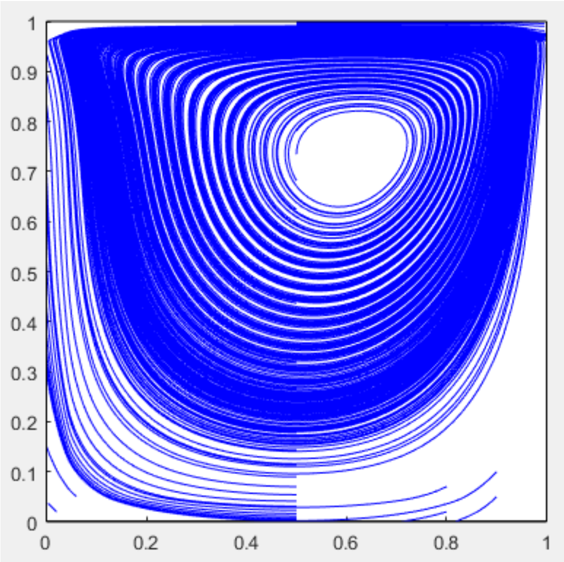

Figure 9 Re=100

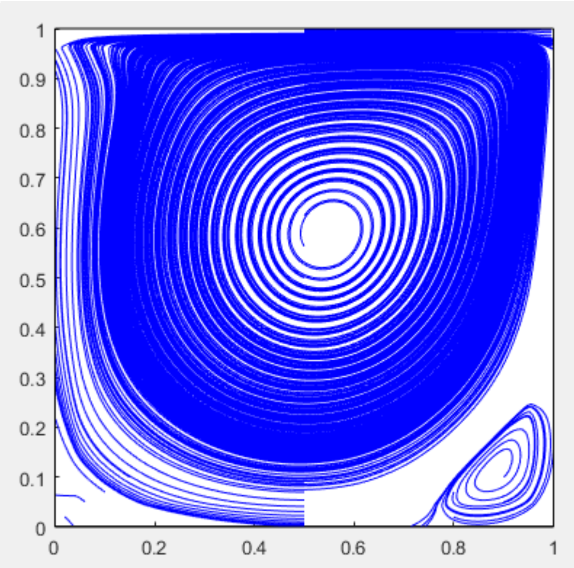

Figure 10 Re=500

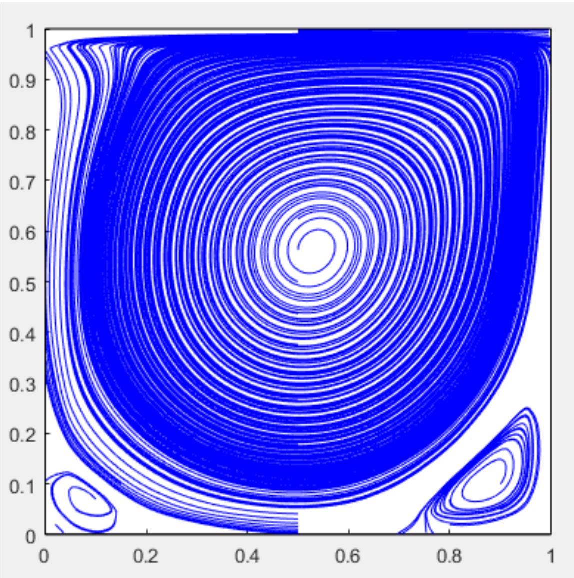

Figure 11 Re=1000

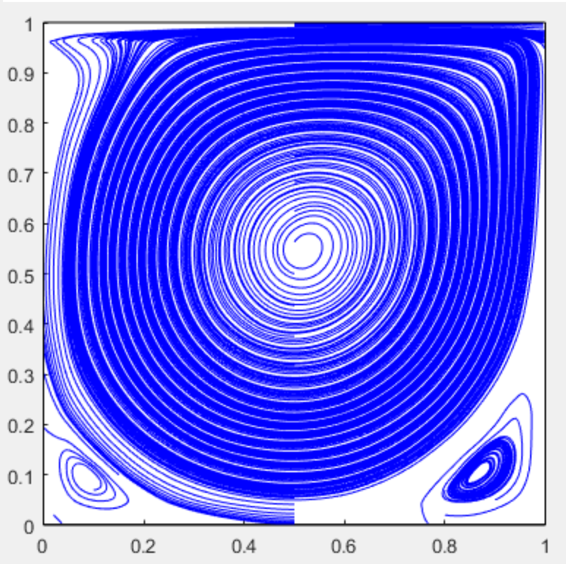

Figure 12 Re=2000

The solution shows that, when the fluid is under the inertia forces, the appearance of vortex on the corners are perceived. Also the number of iterations increase with Reynolds number.