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FEF FINAL EXAM.

1 a) Yes it is suitable because it passes the LBB condition.

b) Yes it is suitable because it guarentees or enforces the positive definiteness of the final matrix obtained from the discrete equations.

c) For DG method we can approximate the pressure with the same polynomial degree as the velocity and the element still fulfills the LBB condition.
The global size of the system :

d) Global problem corresponds only to the red lines : 56 d.o.f.
Local problem corresponds only to the black dots : 10 d.o.f
Velocity global : 56 d.o.f   pressure global (ρ) : 1 d.o.f.
Velocity local : 10 d.o.f   pressure local : 10 d.o.f.
* the above values corresponds only to one direction (x) for example as our problem is in 2D, if considering both direction we just double the number.

e) Assemble and solve the Global problem (loop on the elements and element faces)

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loop on the element to solve the local problem

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loop on the element to post process solution.

2 a) firstly rewritting the equation as follows.

$u_t + k(u) + c(u) + R(u) + \nabla p = f$

$\nabla u = 0$.

$k(u) = -\nu \nabla^2 u$

$c(u) = (u \cdot \nabla) u$

$R(u) = \sigma u$.

time discrete equation:

$$\frac{u^{n+1} - u^n}{\Delta t} + k(u^{n+1/2}) + c(u) + R(u^{n+1/2}) + \nabla p^{n+1} = f^{n+1}$$

$$\nabla \cdot u^{n+1} = 0$$

$$* \; c(u) = [(u \cdot \nabla) u]^{n+1/2}.$$

b) using bilinear & trilinear forms;

$$(w, \Delta u/\Delta t) + a(w, u^{n+1/2}) + c(u^{n+1/2}; w, u^{n+1/2}) + r(w, u^{n+1/2})$$

$$+ b(w, p^{n+1}) = (w, f^{n+1}).$$

$$b(u^{n+1}, q) = 0$$

w - shape function for velocity

q - shape function for pressure.

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c) $$\begin{bmatrix} M & K + C + R & G \\ G & 0 & 0 \end{bmatrix} \begin{bmatrix} u^{n+1} \\ u^{n+1/2} \\ p^{n+1} \end{bmatrix} = \begin{bmatrix} F + Mu^n \\ 0 \end{bmatrix}$$

$$F = \sum_e N_i \cdot f^{n+1}$$

$$M = 1/\Delta t \sum_e N_i \cdot N_j$$
$$K = -\sum_e \nabla N_i \cdot \nabla N_j \cdot \nu$$

$$C = \sum_e N_i \cdot N_j \cdot \nabla N_j$$

$$R = \sigma \sum_e N_i \cdot N_j$$

$$G = \sum_e N_i \cdot Q$$

d) For solving non-linear problems we can use the Picard method. below is the Picard's algorithm.

~~Algorithm~~

$A(u)\,u = b(u)$ (system to be solved).

$$u = A(u)^{-1}\, b(u)$$
$$\Delta u^{k+1} = A(u^k)^{-1}\, b(u^k) - A(u^k)\, u^k$$
$$u^{k+1} = A(u^k)^{-1}\, b(u^k).$$

In our case:

$$A = \begin{bmatrix} M & K + G + R & G \\ G & 0 & 0 \end{bmatrix}$$

$$u = \begin{bmatrix} u \\ p \end{bmatrix}, \quad u = \begin{bmatrix} u^{n+1} \\ u^{n+1/2} \end{bmatrix}$$

$$b = \begin{bmatrix} F + Mu^n \\ 0 \end{bmatrix}$$

this system of equations will be solved on each iteration.

e) for $Re = 100$, both methods are behaving as expected, Picard's methods converges linearly and N-R converges quadratically.

for $Re = 1000$, only the Picard's method is behaving as expected as linear convergence can be observed, but for the N-R method no convergence is observed but some oscillations away from the solution and i think this is so because at $Re = 1000$ the flow is convection dominated and as such highly non-linear which ~~lead~~ might lead to formation of more vortices which causes the N-R method to fail. (for example this can be observed in the cavity flow where another vortex starts to form at the bottom corner of the domain).

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