FINITE ELEMENTS IN FLUIDS

ASSIGNMENT3- 2D STEADY TRANSPORT PROBLEM

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GLS METHOD

Modified the FEM_system to add GLS method. Pretty straight forward by following the theory in slides-

```
elseif method==3
    %GLS
    Pe = a*h/(2*nu);
    tau_p = h*(1 +9/Pe^2)^(-1/2)/(2*a);
    disp(strcat('Recommended stabilization parameter = ',num2str(tau_p)));
    tau = cinput('Stabilization parameter',tau_p);
    if isempty(tau)
        tau = tau_p;
    end
```

For the loop on gauss points, added a reaction variable, set to zero for the initial computation.

```
    elseif method== 3
        reaction=0;
        Ke = Ke + (nu*(Nx'*Nx+Ny'*Ny) + N_ig'*(ax*Nx+ay*Ny) + ...
             ((ax*Nx+ay*Ny)-nu*(Nxx+Nyy)+reaction*N_ig)'...
             *tau*((ax*Nx+ay*Ny)-nu*(Nxx+Nyy)))*dvolu;
        f_ig = SourceTerm(aux);
        fe = fe + (N_ig+tau*(ax*Nx+ay*Ny))'*(f_ig*dvolu);
    end
```

The shape functions provided with the code were incomplete for the double derivates of the quadratic shape functions wrt *xi* and *eta.* The following derivatives were added for p=2 for quadrilateral and triangular elements.

Triangular element

```
    elseif p ==2
     N = [ xi.*(2*xi-1), eta.*(2*eta-1), (1-xi-eta).*(2*(1-xi-eta)-1),...
           4*xi.*eta, 4*eta.*(1-xi-eta), 4*(1-xi-eta).*xi];
     Nxi = [ 4*xi - 1, zeros(size(xi)), 4*eta + 4*xi - 3,...
           4*eta, -4*eta, 4 - 8*xi - 4*eta];
     Neta = [ zeros(size(xi)), 4*eta - 1, 4*eta + 4*xi - 3,...
           4*xi, 4 - 4*xi - 8*eta, -4*xi];
     N2xi = [ 4*ones(size(xi)), zeros(size(xi)), 4*ones(size(xi)),...
           zeros(size(xi)), zeros(size(xi)), -8*ones(size(xi))];
     N2eta = [  zeros(size(xi)), 4*ones(size(xi)), 4*ones(size(xi)),...
           zeros(size(xi)), -8*ones(size(xi)), zeros(size(xi))];
```

Quadrilateral element

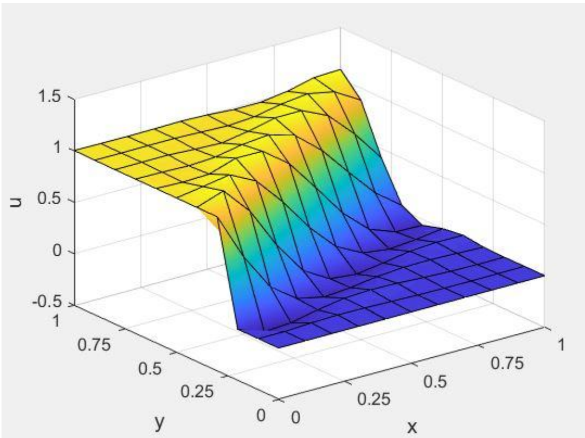
```
    elseif p == 2
        N    = [xi.*(xi-1).*eta.*(eta-1)/4, xi.*(xi+1).*eta.*(eta-1)/4, ...
            xi.*(xi+1).*eta.*(eta+1)/4, xi.*(xi-1).*eta.*(eta+1)/4, ...
            (1-xi.^2).*eta.*(eta-1)/2,  xi.*(xi+1).*(1-eta.^2)/2,   ...
            (1-xi.^2).*eta.*(eta+1)/2,  xi.*(xi-1).*(1-eta.^2)/2,   ...
            (1-xi.^2).*(1-eta.^2)];
        Nxi  = [(xi-1/2).*eta.*(eta-1)/2,    (xi+1/2).*eta.*(eta-1)/2, ...
            (xi+1/2).*eta.*(eta+1)/2,    (xi-1/2).*eta.*(eta+1)/2, ...
            -xi.*eta.*(eta-1),           (xi+1/2).*(1-eta.^2),   ...
            -xi.*eta.*(eta+1),           (xi-1/2).*(1-eta.^2),   ...
            -2*xi.*(1-eta.^2)];
        Neta = [xi.*(xi-1).*(eta-1/2)/2,    xi.*(xi+1).*(eta-1/2)/2, ...
            xi.*(xi+1).*(eta+1/2)/2,    xi.*(xi-1).*(eta+1/2)/2, ...
            (1-xi.^2).*(eta-1/2),       xi.*(xi+1).*(-eta),   ...
            (1-xi.^2).*(eta+1/2),       xi.*(xi-1).*(-eta),   ...
            (1-xi.^2).*(-2*eta)];
        N2xi = [eta.*(eta-1)/2,     eta.*(eta-1)/2, ...
              eta.*(eta+1)/2,       eta.*(eta+1)/2, ...
              -eta.*(eta-1),         (1-eta.^2),   ...
              -eta.*(eta+1),         (1-eta.^2),   ...
              -2*(1-eta.^2)];
        N2eta = [xi.*(xi-1)./2,      xi.*(xi+1)./2, ...
            xi.*(xi+1)./2,           xi.*(xi-1)./2, ...
            (1-xi.^2),              -xi.*(xi+1), ...
            (1-xi.^2),              -xi.*(xi-1), ...
```

Changes made to make u=0 (zero dirichlet boundary) on the outlet boundary, the zero value was assigned to nodes_x1 and nodes_y1 as well.

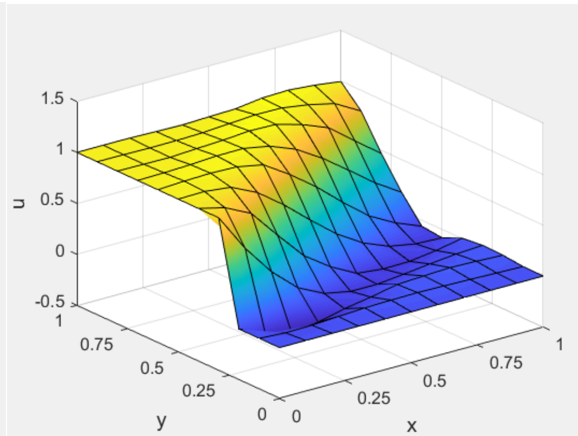
```
% nodes on which solution is u=1
nodesDir1 = nodes_x0( X(nodes_x0,2) > 0.2 );
% nodes on which solution is u=0
nodesDir0 = [nodes_x0( X(nodes_x0,2) <= 0.2 ); nodes_y0];
%for 0 on the outlet boundary
% nodesDir0 = [nodes_x0( X(nodes_x0,2) <= 0.2 ); nodes_y0; nodes_x1; nodes_y1];
```

RESULTS

For quadrilateral and triangular elements, the following results are obtained for p=2(quadratic elements) and 5 nodes.



Quadrilateral element, p=2



Triangular element, p=2

We can see from the results that the code is working for quadratic elements.

Problem variables

The comparisons have been made for both boundary conditions with the following variables

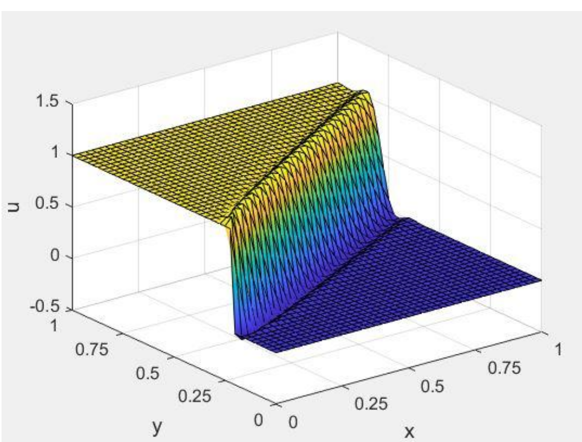
Convection -reaction dominated

||a||=1/2 , $\nu=10^{-4}$, σ=1

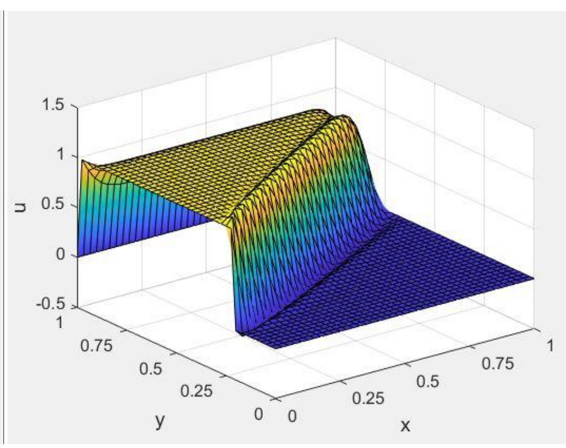
Number of elements in each direction = 20

Quadrilateral element, p=2

Method= GLS



Neumann in outer boundary



Zero Dirichlet on the outer boundary

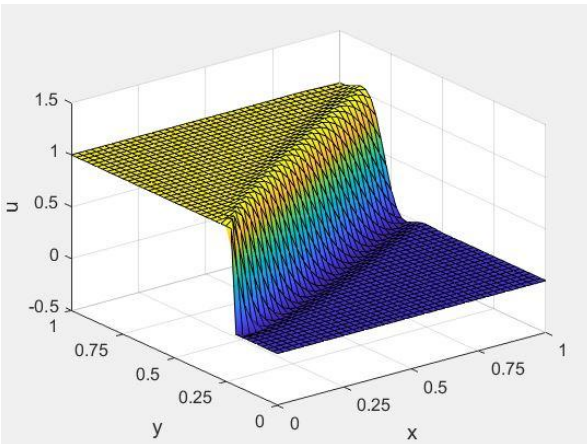
Reaction dominated

$||a||= 10^{-3}$, $\nu=10^{-4}$, $\sigma=1$

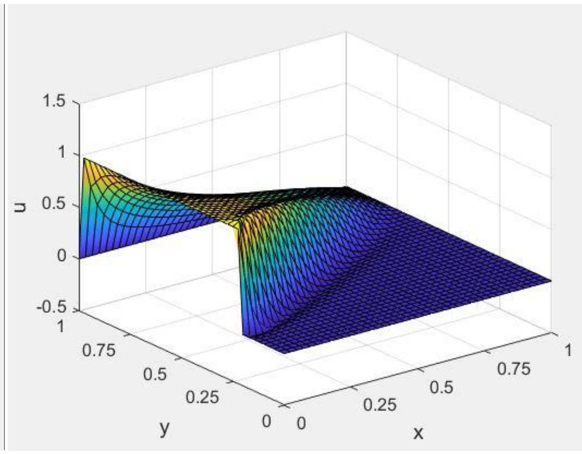
Number of elements in each direction = 20

Quadrilateral element, p=2

Method= GLS



Neumann in outer boundary



Zero Dirichlet on the outer boundary

We can see from both the cases, that the reaction dominated simulation gives a smoother profile than the convection-reaction dominated which is in accordance to the theory. The difference is especially prominent for zero dirichlet boundary where the reaction dominated profile gives a smoother transition to zero velocity compared to convection-reaction dominated.