Lab Report - 4

2D Unsteady Transport problems

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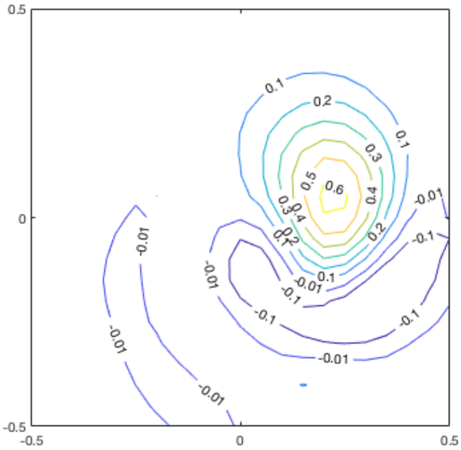
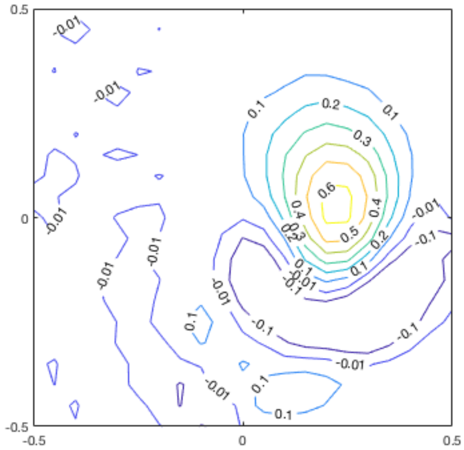
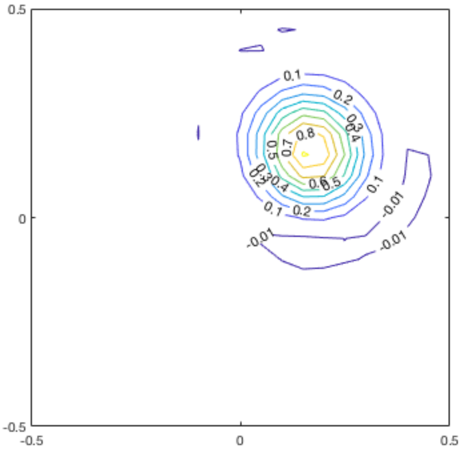
Numerical results are presented for a 20 by 20 elements mesh with a velocity of $(-y, x)$ and where final time is equal to time taken for one complete revolution.



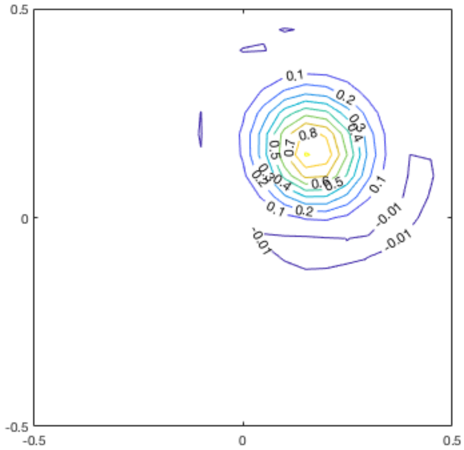
Lax Wendroff + Galerkin Lumped



Crank Nicolson + Galerkin Lumped



TG3 + Galerkin

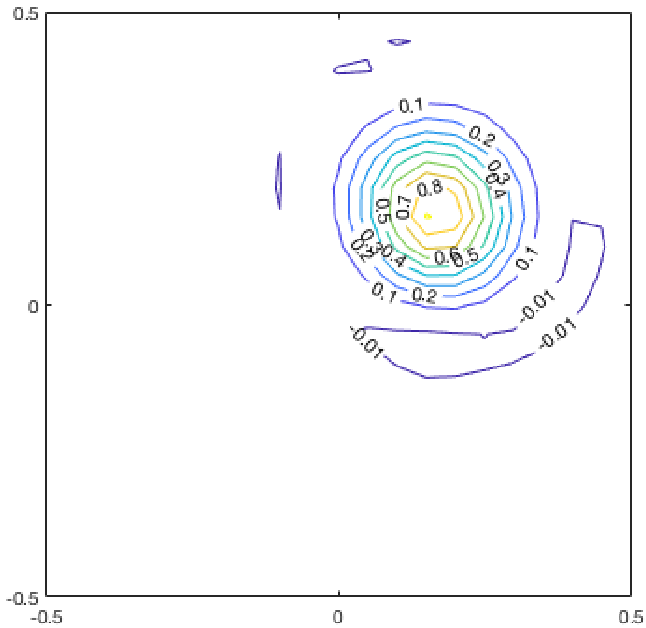


TG3-2S + Galerkin

It can be observed that lumped crank nicolson and lumped lax wendroff produce highly diffusive inaccurate solutions as compared to the higher order methods. The 4th order two step taylor galerkin method seems to be the most accurate choice but is conditionally stable.. The two step fourth order taylor galerkin method is given by

$$\begin{aligned} \tilde{u} &= u^n + \frac{1}{3}\Delta t u_t^n + \frac{1}{12}\Delta t^2 u_{tt}^n \\ u^{n+1} &= u^n + \Delta t u_t^n + \frac{1}{2}\Delta t^2 \tilde{u}_{tt}^n \end{aligned}$$

```
elseif meth == 8 % TG4 - 2S
    A1 = M;
    B1 = (1/3)*C*dt - (1/12)*K*dt^2 - (1/3)*Mo*dt + (1/12)*Co*dt^2;
    f1 = (1/3)*v1*dt + (1/12)*v2*dt^2 - (1/12)*vo*dt^2;
    A2 = M;
    B2 = C*dt - Mo*dt;
    f2 = v1*dt + 0.5*v2*dt^2 - 0.5*vo*dt^2;
    C2 = -0.5*K*dt^2 + 0.5*Co*dt^2;
```



TG4-2S + Galerkin

Element matrices and vectors are given by

$$
\begin{aligned}
C &= (a.\nabla w, u) \\
K &= (a.\nabla w, a.\nabla u) \\
v_1 &= (s, w) \\
v_2 &= (a.\nabla w, s) \\
C_0 &= a.n(w, a.\nabla u) \\
M_0 &= a.n(w, u) \\
V_0 &= a.n(w, s)
\end{aligned}
$$