Finite element in fluid

Assignment 3

Author : Seyed mohammadreza Attar Seyedi

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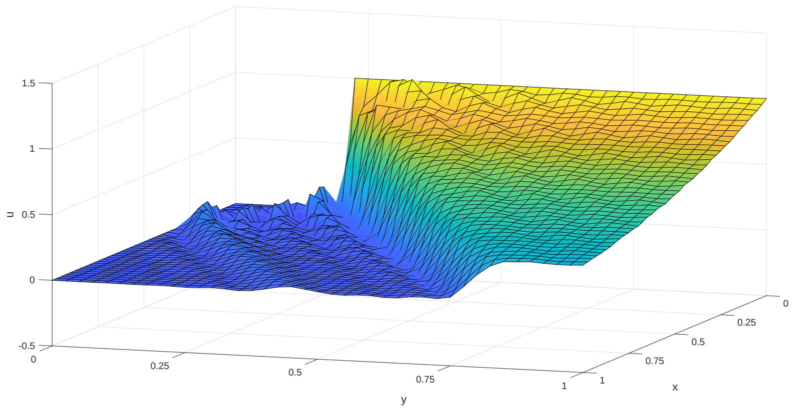

Figure 1: Galerkin

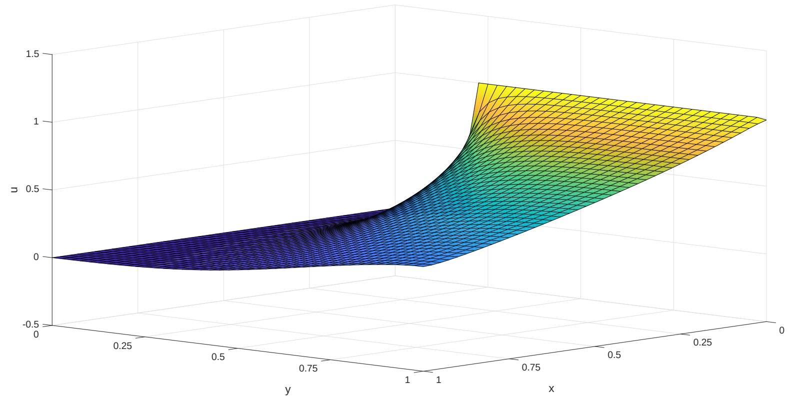

Figure 2: Artificial

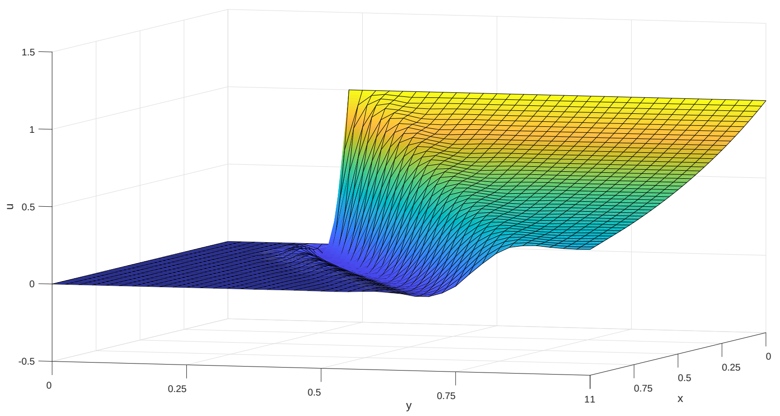

Figure 3: SUPG

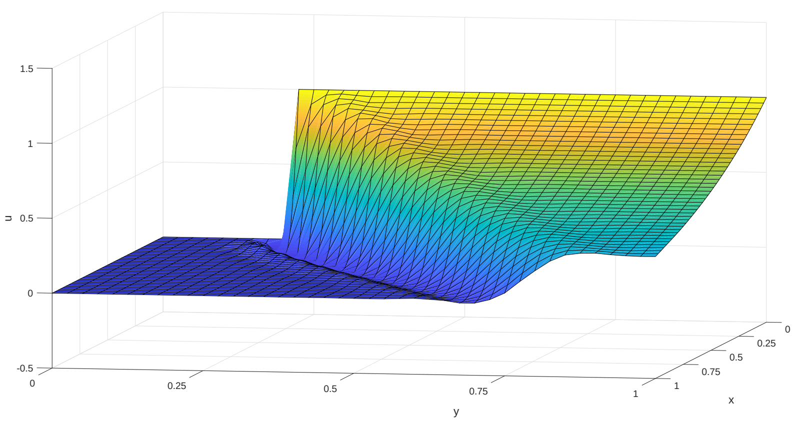

Figure 4: GLS

Homogenous natural boundary conditions:

$||a|| = 1$ and the convection velocity is skew to the mesh with an angle of $30°$ and $\nu = 10^{-4}$. when the Peclet number is very high, the solution is practically pure convection.
Galerkin method is not good for resolve the discontinuity and produces spurious oscillations.
Artificial diffusion introduces too crosswind diffusion.
The Artificial diffusion method and SUPG yield better results, but SUPG introduction less crosswind diffusion.

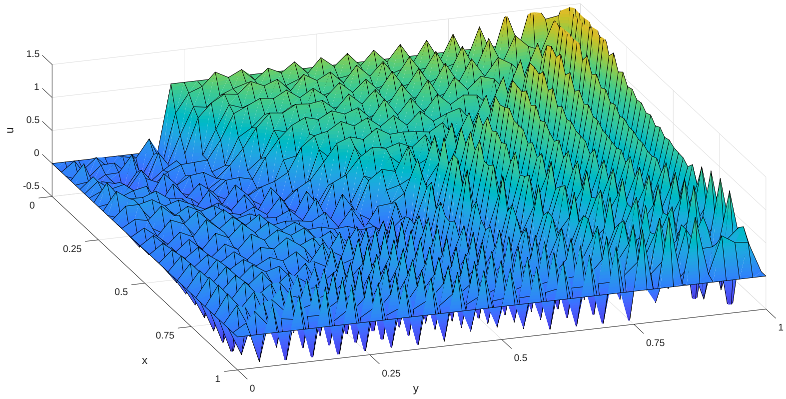

Figure 5: Galerkin

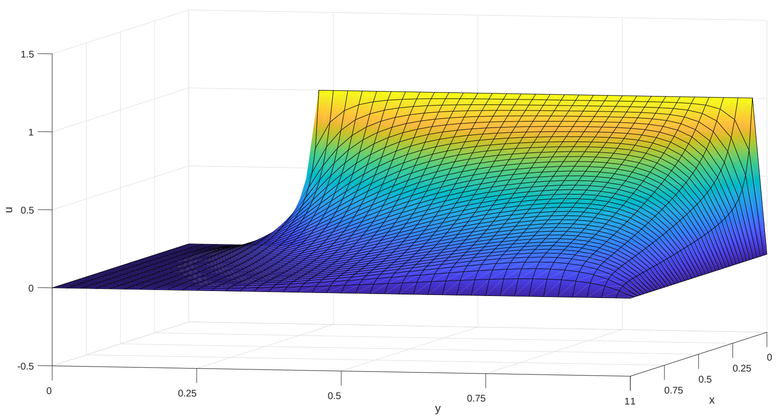

Figure 6: Artificial

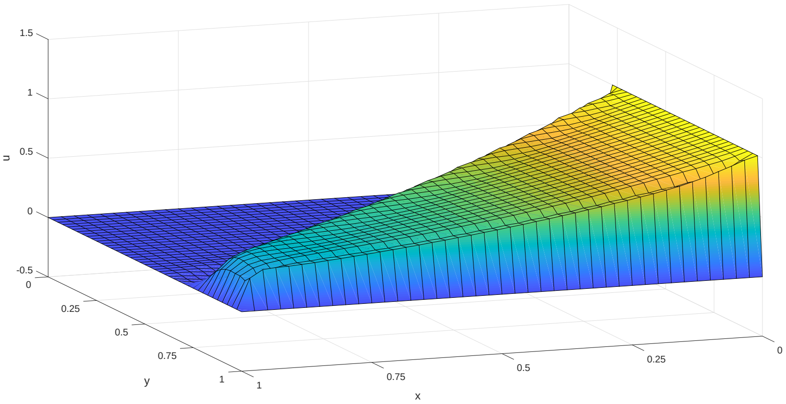

Figure 7: SUPG

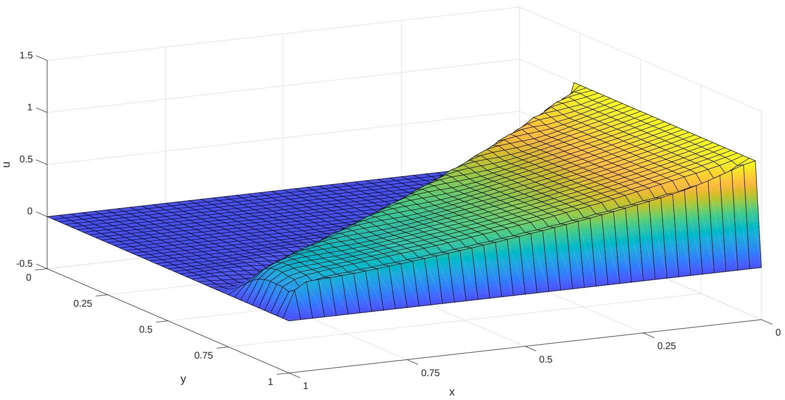

Figure 8: GLS

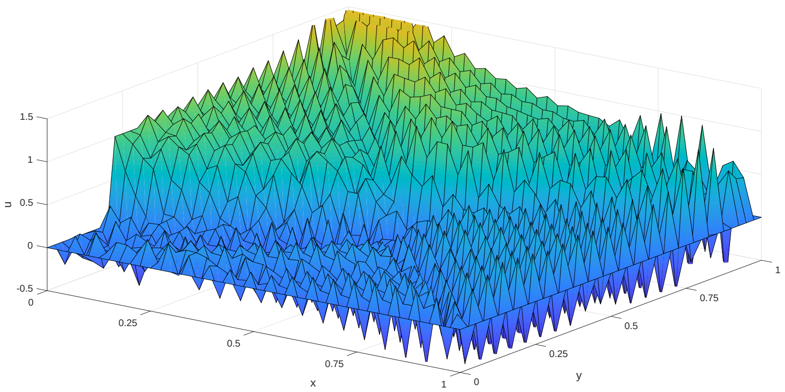

Figure 9: Galerkin

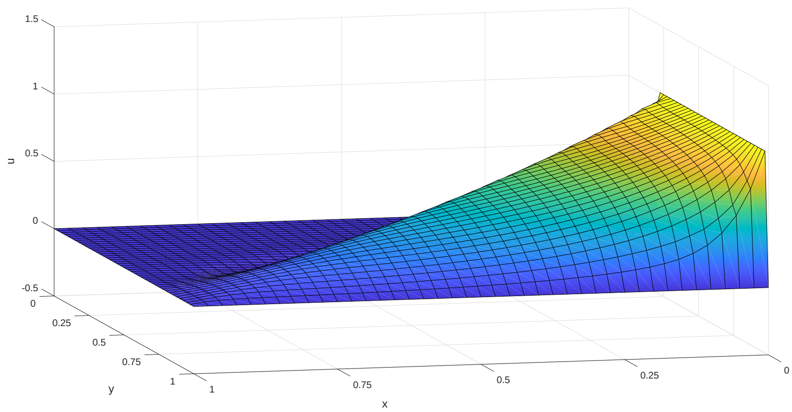

Figure 10: SUPG

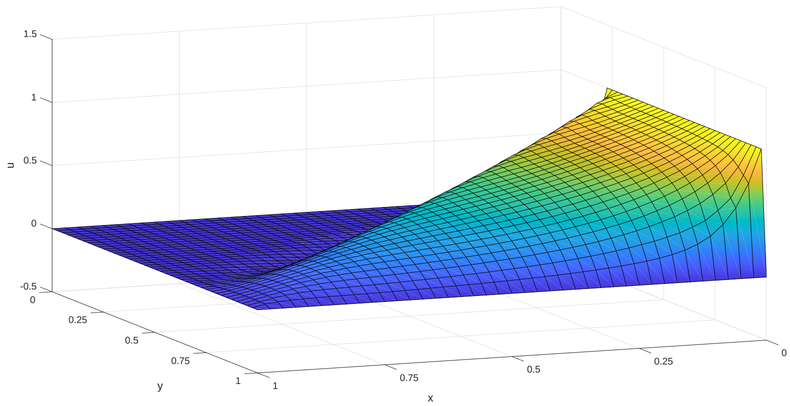

Figure 11: GLS

Zero Dirichlet condition :

2D case has been widely used to illustrate the effectiveness of stabilized finite element methods in convection.
The solution involves a thin boundary layer at the outlet portion of the boundary.
The Galerkin results are widely oscillatory and bear no resemblance to the exact results.
Artificial diffusion is based on a scalar diffusivity.
The Artificial diffusion introduces excessive numerical diffusion.
Better results are obtained with the stabilization parameter.
The influence of the convection term is computed for Galerkin, SUPG and GLS formulation.
Convection is taken skew to the mesh with an angle 30° and $\nu = 10^{-4}$.
The stabilization methods produce similar results.
SUPG and GLS produce good solution.

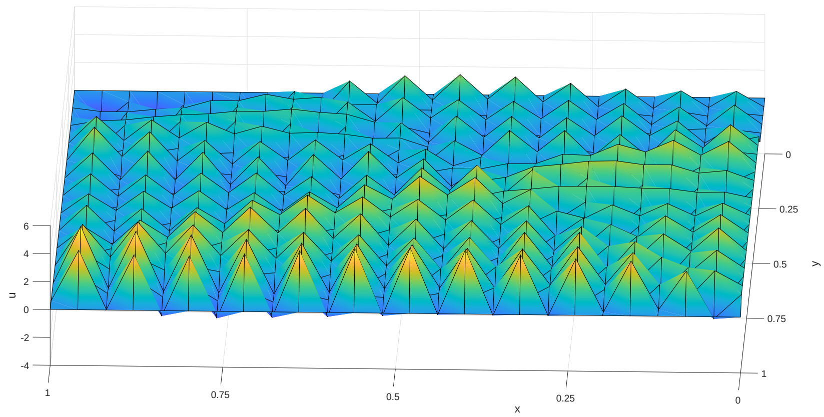

Figure 12: Galerkin

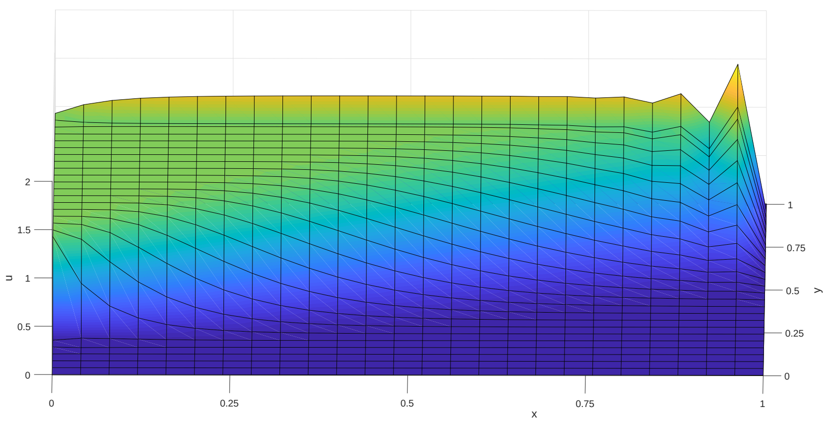

Figure 13: Artificial

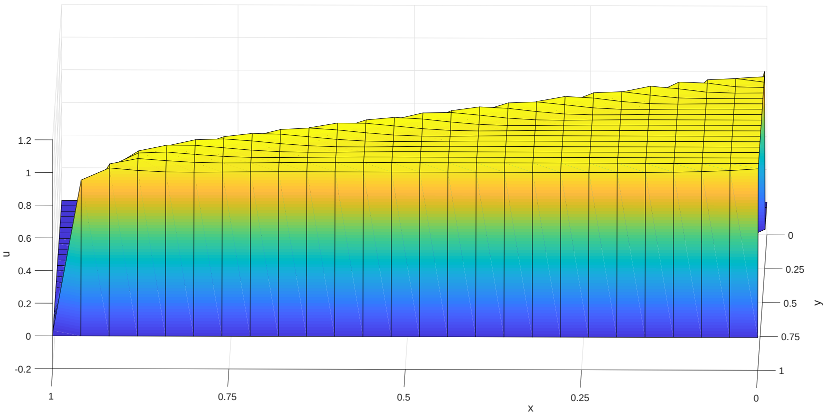

Figure 14: SUPG

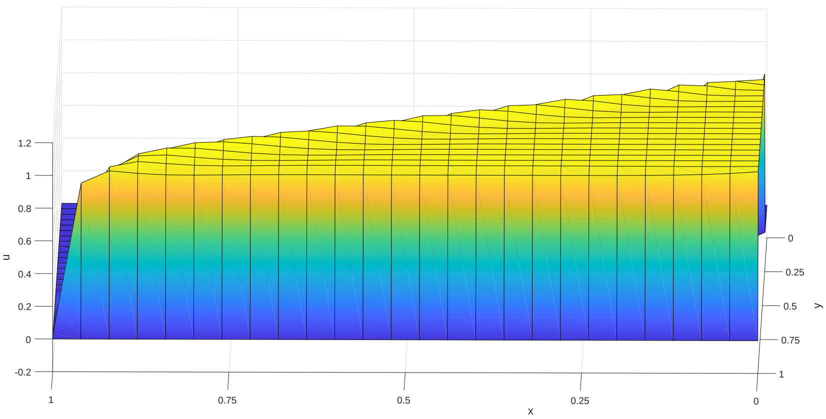

Figure 15: GLS

Convection is taken skew to the mesh with an angel $30°$. $||a|| = 10^{-3}$ and $\sigma = 1$.

Annex

```
ifmethod == 0
Galerkin
tau = 0;

elseifmethod == 1
ArtificialDiffusion
Pe_x = ax * h/(2 * nu);
Pe_y = ay * h/(2 * nu);
alpha_x = coth(Pe_x) - 1/Pe_x;
alpha_y = coth(Pe_y) - 1/Pe_y;
tau = h * (ax * alpha_x + ay * alpha_y)/2;
nubar_p = tau * a * a;
nubar = cinput('Artificialdiffusiontobeused', nubar_p);
ifisempty(nubar)
nubar = nubar_p;
end
nu = nu + nubar;
tau = 0;

elseifmethod == 2
SUPG
Pe = a * h/(2 * nu);
tau_p = h * (1 + 9/Pe^2)^( - 1/2)/(2 * a);
tau = cinput('Stabilizationparameter', tau_p);
ifisempty(tau)
tau = tau_p;
end

elseifmethod == 3
GLS
Pe = a * h/(2 * nu);
tau_p = h * (1 + 9/Pe^2)^( - 1/2)/(2 * a);
tau = cinput('Stabilizationparameter', tau_p);
ifisempty(tau)
tau = tau_p;
end
```

$if method == 0$
$Galerkin$
$Ke = Ke + (nu * (Nx' * Nx + Ny' * Ny) + N_ig' * (ax * Nx + ay * Ny) + N_ig' * sigma * N_ig) * dvolu;$
$aux = N_ig * Xe;$
$f_ig = SourceTerm(aux);$
$fe = fe + N_ig' * (f_ig * dvolu);$

$elseif method == 1$
$Artificial diffusion$
$Ke = Ke + (nu * (Nx' * Nx + Ny' * Ny) + N_ig' * (ax * Nx + ay * Ny) + N_ig' * sigma * N_ig) * dvolu;$
$aux = N_ig * Xe;$
$f_ig = SourceTerm(aux);$
$fe = fe + N_ig' * (f_ig * dvolu);$

$elseif method == 2$
$SUPG$
$Ke = Ke + (nu * (Nx' * Nx + Ny' * Ny) + N_ig' * (ax * Nx + ay * Ny) + N_ig' * sigma * N_ig + tau * (ax * Nx + ay * Ny)' * ((ax * Nx + ay * Ny) - nu * (N2x + N2y) + sigma * N_ig)) * dvolu;$
$aux = N_ig * Xe;$
$f_ig = SourceTerm(aux);$
$fe = fe + (N_ig + tau * (ax * Nx + ay * Ny))' * (f_ig * dvolu);$

$elseif method == 3$
GLS
$Ke = Ke + (N_ig' * (ax * Nx + ay * Ny) + nu * (Nx' * Nx + Ny' * Ny) + N_ig' * sigma * N_ig + tau * ((ax * Nx + ay * Ny) - nu * (N2x + N2y) + sigma * N_ig)' * ((ax * Nx + ay * Ny) - nu * (N2x + N2y) + sigma * N_ig)) * dvolu;$
$aux = N_ig * Xe;$
$f_ig = SourceTerm(aux);$
$fe = fe + (N_ig + tau * ((ax * Nx + ay * Ny) - nu * (N2x + N2y) + sigma * N_ig))' * (f_ig * dvolu);$
end